1 Introduction

The recent financial market turmoil has raised the question whether certain countries, in particular Italy and Spain, were facing a liquidity or a solvency problem. In the former case, but not in the latter it was argued the ECBs intervention in the sovereign bond market, or even the simple commitment to do so, could put the situation under control. Some economists (e.g., Paul Krugman (2011)) prefer to rephrase this problem, by asking whether those countries crisis was triggered by investors self-fulfilling expectations which are not, or not entirely, justified by fundamentals. This phenomenon - sometimes labeled self-fulfilling insolvency (or illiquidity) trap - is characterized by two related, perverse features: the presence of upward-sloping tracts of the demand for bond function and the associated multiple equilibria. All this is reminiscent of earlier discussions concerning the crisis (and collapse) of the Exchange Rate Mechanism of 1992-1993 (see, for example, Eichengreen and Wyplosz (1993), Portes (1993), Obstfeld (1994, 1996) and Krugman (1996)), as well as the emerging countries currency crises of the late 1990s (see for example, Masson (1999) and the references therein). In all these cases, the variables in question might be different but the theoretical issues are essentially the same. A number of recent, macroeconomic models focusing on the euro sovereign debt crisis have been produced (cf. De Grauwe (2011a 2011b)) that allow for multiple equilibria and self-fulfilling mechanisms.

A simple, suggestive graphical representation of the main point in question can be found in Ip (2011), which is reproduced in Figure 1 below.\footnote{Notice that in Ip’s diagram axes are interchanged with respect to those in our diagrams below.}

The logic of these models would require an explicit dynamical analysis of the bond market that, in the relevant literature, is often cursory or altogether missing, while some crucial assumptions are introduced only implicitly. The attention is entirely concentrated on equilibria, multiple or otherwise, and their stability properties and the possibility of more complex and interesting modes of behavior is altogether neglected. In this paper we would like to make a contribution to filling this gap in the literature.
2 The standard model

The phrase “standard model” in this section must be taken with a grain of salt and needs some preliminary considerations.

Trying to make sense of the verbal and graphical suggestions found in the recent literature and specialized press concerning the problem of insolvency traps and multiple equilibria in relation to sovereign bond markets, a broad characterization of a standard model of price dynamics for a single bond market could be described as follows:

- (i) While in textbook discussions of bond markets it is typically assumed that in a bond market the demand and supply curves as functions of price have, respectively, a negative and a positive slope, in the special case we are discussing here, it is assumed that, over certain ranges of value of the bond price, its demand curve may have a “perverse”, positive slope, whereas the supply curve is always upward sloped.

- (ii) As a consequence of (i), there may exist multiple equilibria of the bond market. Typically, it is assumed that the demand curve has a S-shaped form and that there are three intersections with the supply curve, i.e., three equilibria, namely: a “bad” equilibrium, $E_1$, characterized by low price/high yield; a “good” equilibrium, $E_2$, with high price/low yield; an intermediate one, $E_0$.

- (iii) At $E_1$ and $E_2$, the demand curve has a “regular”, downward slope while it has an upward slope at $E_0$. The supply curve has an upward slope.

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2Demand for and supply of a bond can be defined, equivalently, as functions of its price or its yield. Other things being equal, each of these variables is determined when the other is known and they move in opposite directions. In the mathematical formulations that follow, we use price as the state variable. To fix ideas, the reader may assume that we are discussing a zero coupon bond.

3Actually, there is nothing perverse or illogical in an upward sloping demand curve. It often occurs when a consumer, lacking a complete information about the quality of certain goods whose purchase s/he is considering, takes price as an element to evaluate its quality. Similarly, an incompletely informed investor may take the high price of a bond as a market signal that it is less risky (or, inversely, a high yield as a sign that it is riskier). Thus, other things being equal, the perceived lower risk of a bond signaled by a higher price, or a lower yield, tends per se to increase the demand for it – and vice versa for a lower price (higher yield). Whenever this effect, call it “risk effect”, prevails on the “normal” substitution effect, the demand curves slopes upward.

4Notice that the choice of an S-shaped curve implies the assumption that the substitution effect prevails over the risk-effect for very low and very high prices and the opposite is true for intermediate ones.
slope everywhere. Hence, in a neighborhood of the $E_1$ or $E_2$ equilibrium, prices higher (lower) that the equilibrium value correspond to a negative(positive) excess demand. The opposite is true for the intermediate equilibrium $E_0$.

- (iv) The bond price changes as a function of the excess demand: it increases when excess demand is positive and decreases in the opposite case.

- (v) From (iii) and (iv), it follows that the equilibria $E_1$ and $E_2$ are (locally) stable and $E_0$ is unstable. In particular, starting for any initial price lower than that corresponding to $E_0$, $p_0$, the market will converge to the “bad” equilibrium $E_1$; vice versa, starting from any initial price above $p_0$, the market will converge to the “good” equilibrium $E_2$.

Remark 1. In our discussion so far there is a mostly implicit reference to the effects of the level of price/yield of sovereign bonds and the solvency of the issuer. The concept of solvency/insolvency, its relation with that of liquidity/illiquidity is a complex one and even a preliminary discussion of the related problems will take us afar. For our present purpose, it will be sufficient to make use of a broad, easily evaluated formula establishing the macroeconomic condition for a sovereign country’s solvency, in the limited sense that when the condition is verified the country’s Debt/GDP is not increasing or decreasing. In macroeconomic debates, this is usually taken as a condition for the sustainability of debt.

Let $s$ denotes primary budget surplus, i.e. surplus net of interests; $r$ is the yield of the “representative” sovereign bond; $g$ is the rate of growth of GDP and $d$ is the initial debt/GDP ratio, and let us write:

\[ s \geq (r - g)d \]  

(1)

It is easy to show that if the above inequality is satisfied, the debt/GDP ratio is not increasing, or, if there is a strict inequality ($>$ rather than $\geq$) that ratio is decreasing. This is an *ex-post* condition and it does not take into account the complex (and controversial) relation between its components $s, r, g$. However, the formula clearly indicates that, since a significant increase in the primary surplus and the rate of growth are difficult to realize, all the more so in the short run, a low and sustained level of price (a high and sustained level of yield) may quickly lead to financial collapse. Moreover, the bond price and the corresponding yield rate largely depend on investors’ expectations which can easily become self-fulfilling: a low price (high yield) based on an unwarranted pessimistic view of a country’s solvency could generate
an ex–post justification of those expectations, leading to a further decrease in price (a further increase in yield) of the bond, and so on and so forth in a self–sustaining perverse dynamics.

This super–simplified, but not entirely unreasonable representation of the bond market, could be (and has been) used to provide some support for certain measures of economic policy. Consider for example a country and its ‘representative sovereign bond’ (say the Italian 10 year BTP), for which, other things being equal, a high price (low yield) of the price corresponding to $E_2$ would guarantee financial stability (a non-increasing debt/GDP ratio), whereas a low price (high yield) corresponding to $E_1$ would be unsustainable and lead to financial collapse. In this case, an exogenous intervention pushing the bond price above the critical level $p_0$ would do the trick, because at that point a spontaneous “virtuous dynamics” would start and, if the other parameters do not change, would lead to sustainability of the debt.

### 3 Critique of the standard model and summary of results

Unfortunately, the conclusions drawn from the standard model, even in the case in which only one, “representative” bond is considered, do not hold in general, at least not if a model describing the bond price dynamics is formulated in a discrete–time form. The basic reason is that proposition (v) above is not valid for this type of models. Even if the price reacts “correctly” to excess demand and, starting from a value off equilibrium, moves towards it, the correction can be excessive, there can be an overshooting and, if the latter is very strong, unstable and more or less complex oscillations may follow. We return to this point in a greater detail later.

**Remark 2.** The implications of the choice between discrete– and continuous–time in economic dynamical models are seldom discussed thoroughly in economic literature, least of all in the recent discussions on multiple equilibria. At any rate, we don’t know of any cogent argument proving that one or the other way of treating time in economic models is generally preferable. Here, we limit ourself to notice that the dynamics of single variable continuous–time models is severely restricted for well–known mathematical considerations. In that case, whatever the other specifications of the model may be, only convergence to, or divergence from a fixed point (equilibrium) is possible. Making use of economists’ terminology, this fact implies that in a single variable, continuous–time model there cannot be “overshooting. Once the variable
moves towards an equilibrium, absent external shocks, it will inevitably converge to it. Vice versa, once it starts moving away from an equilibrium, it will never converge back to it (although it might converge to another one. This is no longer true, however, if a second state variable is considered. In this case, the state of two–variable system, defined by a pair of values of the two variables at a given time, may alternatively move towards an equilibrium and away from it without ever converging to a stationary state. It may instead converge to a cycle around the equilibrium. Ever here, however, the possible types of dynamics are restricted for mathematical considerations analogous to those mentioned above. To see this, consider the case of an unstable equilibrium surrounded by a unique, globally stable cycle. The latter defines a closed curve in the two–dimensional space for which there is an “inside” and an “outside”. Then, excluding the very special cases in which the initial state is located on the equilibrium or on the cycle, there are only two possible modes of behavior: either the system converges to the cycle – moving from inside or from outside – or it diverges from it, but no crossing of the cycle is permitted. Thus, in the economic parlance, we might say that although there can be “overshooting of the equilibrium” (along the cycle the distance from equilibrium may alternatively decrease and increase), there cannot be “overshooting of the cycle”. If time is treated as a continuous variable, and do not want to preclude a priori the possibility of more complex or chaotic dynamics, we must deal with models with no less that three variables. For the present discussion, it is interesting to recall that early results of economic dynamics have shown that general economic equilibrium models where price movements are governed by the “law of demand and supply”, equilibria are not necessarily stable and there may be cyclical or more complex dynamics.

In conclusion: if we want to keep our model simple and manageable and deal with a single market for a representative bond, and at the same time we don’t want to assume away from the beginning more complicated and interesting mode of behavior, the choice of a discrete–time setting seems the most reasonable.

In this paper we show that, even for the extremely simplified representation of a bond market provided by the standard model, things may be be significantly different from and greatly more complicated than we are often led to expect. First of all, even in the most favorable circumstances – a single equilibrium and a downward–sloping demand curve, the equilibrium may not be stable and the state variables may converge to a cycle around it, and more or less far from it. Thus, even if this unique equilibrium is “good” in the sense that the corresponding price(yield) is compatible with a country’s solvency, the average price(yield) along the stable cycle may be too high to stabilize the debt, given the other fundamental parameters. Second, if the single
equilibrium is unstable and, due to the presence of upward-sloping tracts of the demand curve, the controlling map is non-monotonic, the asymptotic dynamics of the bond’s price (and the corresponding yield) may be very complicated (cyclical with different periods or even chaotic). In the multiple equilibria case, many different, more or less complicated dynamics may occur. We have followed the literature and adopted a “S-shaped” demand function. In this case, we may have three equilibria (a “good” one, characterized by high price/low yield, a “bad” one, with low price/high yield and an intermediate one, which is always unstable. When the other two equilibria are stable, as it is commonly assumed, the intermediate equilibrium may play the role of a watershed between the basins of attraction of the “good” and the “bad” equilibria, defining, respectively a “safe” and “unsafe” interval of price (yield) values of the bond. Therefore, an external intervention (in the form of the central bank’s purchase of the bond or simply its commitment to that action) can push the bond’s price (yield) into the safe zone, thus stabilizing the market. However, this need not be the case: as we show, either the “good”, or the “bad” equilibrium, or both, may be unstable and complicated dynamics are possible. Asymptotically, the market may oscillate cyclically or chaotically around one or the other of the two equilibria the market, even in the absence of shocks. In this case, an external push, moving the bond’s price up (the bond’s yield down), close as we wish to its “good” equilibrium value will not be sufficient to stabilize the market, and sustainability may require a continuous support. In the case of a single, “good” or “bad” equilibrium, other intriguing possibilities may arise, the most interesting of which is known to mathematicians as “intermittency”, characterized by potentially very long periods of tranquility with price (yield) values arbitrarily close to equilibrium, followed by more or less violent fluctuations - a phenomenon that we would be tempted, prima facie, to explain by external shocks that in our case do not exist.

4 A mathematical model of a bond market

Consider a financial market where a single bond issued by a sovereign state is exchanged at a price \( p \). Demand for and supply of the bond are functions of the price and denoted, respectively, \( D(p) \) and \( S(p) \). Economic “first principles” and empirical investigations provide us with rather broad suggestions for choosing specific functional forms for the two functions. We adopt the common hypothesis of a S–shape demand function. There are several options compatible with this hypothesis, but we have found that a third degree polynomial (with a conveniently restricted domain) provides a mathematical
tool sufficiently flexible to analyze a variety of possible dynamical behavior. We also choose a logarithmic supply function monotonically increasing at a decreasing rate.

The bond price changes at discrete intervals $\Delta$ of time, $t$, of unspecified length. Taking $\Delta$ as the arbitrary unit of measure of time, $t$ is defined as an integer, i.e., $t = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$ where 0 denotes the present time, negative integers are the past and positive ones the future. The value of the price at a generic time $t$ is denoted by $p_t$. The price changes according to the “law of demand and supply”, i.e., it increases when demand exceeds supply and vice versa. In simpler form this may be rephrased by saying that the price increases, decreased or remains constant, according to whether excess demand $ED(p) = D(p) - S(p) \geq 0$. From these definitions and hypotheses we can define the following dynamical system in the variable $p$ (the time index is omitted when unnecessary):

$$D(p) = a_0 - a_1 p + a_2 p^2 - a_3 p^3 \quad (2)$$

$$S(p) = \ln(p + k) \quad (3)$$

$$ED(p) = D(p) - S(p) \quad (4)$$

$$p_{t+1} = p_t + ED(p_t) = G(p_t) \quad (5)$$

where $a_i, i = 0, 1, 2, 3$ and $k$ are positive parameters and $p \geq 0$. If we assume that a minimum, strictly positive value of $p$ is required to induce a nonnegative supply of the bond, then it must be $k < 1$.

**Remark 3.** Dimensional considerations. Broadly speaking, the dimension of price $p$ is of the type money/quantity (e.g. $/q$, where $\$ is the US Dollar and $q$ denotes the unit in which demand ans supply of a bond are measured), while the dimension of the quantities $D$ and $S$ is $q$. Therefore, strictly speaking, dimensional compatibility of the terms of equation (5) would require that the term denoting excess demand be multiplied by a parameter, say $\theta$, with dimension $p/q = $$/q^2$, where $\theta$ can be interpreted as the strength of the reaction of the price to a given value of excess demand. For simplicity’s sake, we assume here that $\theta = 1$ and omit to write it.

5 A short refresher on dynamics of one–dimensional maps

The dynamics of the bond price $p$ and by implication that of its yield, is entirely determined by the properties function $G(p_t)$ which in this context is usually called “map”. Starting from any arbitrary initial value of $p$, the
iterations of $G$ will determine the history of $p$ “from here to eternity”. Equilibria of the dynamical system described by (5) correspond to fixed points of the map $G$, i.e. values $\bar{p}$ of $p$ such that $G(\bar{p}) = \bar{p}$, i.e. $ED(\bar{p}) = 0$, or in economics terms, equality of demand and supply. A fixed point of $G$ is (locally) stable or unstable if

$$|G'(\bar{p})| = |1 + D'(\bar{p}) - S'(\bar{p})| = 1 + ED'(\bar{p}) \leq 1$$

Thus, stability depends essentially on $ED'(\bar{p})$ and thereby on $D'(\bar{p})$ and $S'(\bar{p})$. Assuming that $S'(\bar{p}) > 0$ always, here are the main cases and sub–cases.

1. $G'(\bar{p}) > 0 \Rightarrow ED'(\bar{p}) > -1$. No overshooting of equilibrium; two sub–cases:
   (a) $0 < G'(\bar{p}) < 1 \Rightarrow ED'(\bar{p}) < 0$. Stable equilibrium.
   (b) $G'(\bar{p}) > 1 \Rightarrow ED'(\bar{p}) > 0$. Unstable equilibrium.

2. $G'(\bar{p}) < 0 \Rightarrow ED'(\bar{p}) < -1$. Overshooting of equilibrium; two sub–cases:
   (a) $-1 < G'(\bar{p}) < 0 \Rightarrow ED'(\bar{p}) > -2$. Oscillations converging to equilibrium.
   (b) $G'(\bar{p}) < -1 \Rightarrow ED'(\bar{p}) < -2$. Oscillations diverging from equilibrium.

Given the simplicity of the model in question, the dynamics of the state variable $p$ can be easily visualized by means of the so–called “cob–web” diagram. In a Cartesian system of axes, we put $p_{t+1} = G(p_t)$ on the ordinate and $p_t$ on the ordinate and, in the positive quadrant, we plot the graphs of $G$ and the bisector (Bis($p$) = $p$). Then the intersections of the two curves correspond to equilibria. Stability of each equilibrium depends on whether the (absolute value of the) slope of $G$ at the intersection is greater or smaller than 1. This may be immediately established when $G'(\bar{p}) > 0$, and may be guessed when $G'(\bar{p}) < 0$. Off–equilibrium orbits starting from any initial $p = p_1$, the next value $p_2$ may be determine by drawing a vertical segment (up or down) to the curve $G(p)$ and from there an horizontal segment (rightward or leftward) to the bisector. Iterating this procedure, one can gets all the subsequent values of $p$. In the simplest cases, a few iterations are sufficient to establish the dynamic pattern of the orbits.

A few points should be stressed here.

* (i) Assuming that $S'(p) > 0$ always, a “perverse”, upward–sloping demand curve, i.e. $D'(p) > 0$ is a necessary condition for instability of type 1b equilibria (without oscillations).
• (ii) $D'(p) > 0$ is also a necessary condition for the existence of multiple equilibria.

• (iii) However, instability of type 2b (with oscillations) is perfectly compatible with a “normal”, downward-sloping demand curve. In words, this case occurs when traders’ excessive response to off-equilibrium prices yields self-reinforcing market overcorrection. This type of instability may occur independently of the existence of multiple equilibria.

• (iv) In the model we are describing, whether there is one or many equilibria, all of them may be unstable.

• (v) Contrary to what we are sometimes led to believe, there is no guarantee that if the initial value of the bond price starts above (below) the watershed value $p_0$, it will stay there. For example, as we will see below, it is possible for $p$ to start arbitrarily close to $E_2$ and converge eventually to $E_1$.

These points will be illustrated by the analysis that follows. We will see that when the state variable $p$ moves away from an unstable equilibrium it does not necessary converge to another, stable one but may follow more or less complicated orbits (cyclical or chaotic) while remaining within acceptable ranges of values. However, in some cases, the values of $p$ may cease to make economic sense (e.g. becoming negative or excessively large) and we shall say then that “the system explodes”.

The properties of the price(yield) dynamics generated by the map $G$ clearly depend on the numerical values of the structural parameters of $D$ and $S$. Unfortunately, from economic “first principles” we can derive only very broad criteria to constrain the excess demand function and consequently the choice of parameters is excessively large. Our aim here is to show that there exist not obviously unreasonable combinations of parameters for which the dynamics of $p$ is much richer and interesting that we sometimes are led to believe.

6 The dynamics of a bond market

6.1 Locally stable “good” and “bad” equilibria

We begin with two cases for which the conclusions of the standard model hold (see, in particular the point 1.(v) above). The first one, the “pure standard case”, is illustrated by Figure 2.
There are two locally stable equilibria, both of type 1a, \( E_1, E_2 \) and a third, unstable of type 1b, \( E_0 \). The equilibrium price corresponding to \( E_0 \), \( p_0 \) defines two regions of the state space: orbits originating from \( p < p_0 \) lead to the "bad" equilibrium \( E_1 \) and financial collapse; orbits starting from \( p > p_0 \) (within appropriate limits) lead to the "good" equilibrium \( E_2 \) and solvency. No overshooting or oscillations occur.

The second case, "the modified standard case" is illustrated by Figure 3.

It is analogous to the first one with the difference that the two stable equilibria \( E_1 \) and \( E_2 \) are now of type 2a. Consequently, there will be overshooting and convergence to equilibrium will take place by way of damped oscillations. When the slope of the excess demand curve at equilibrium increases, \( E_1 \) and/or \( E_2 \) may became unstable with more or less unfavorable consequences.

### 6.2 The "good" equilibrium is unstable. All equilibria are unstable

Things need not be always as simple as in 6.1. We only mention a few interesting sub-cases. A simple occurrence is illustrated by Figure 4 where stable period-2 cycles appear around \( E_2 \).

It would be easy to produce cases yielding attracting cycles of higher periods. Instability of an equilibrium may lead to much more complicated dynamics. Figure 5 shows the case for which the dynamics around \( E_2 \) suggests the existence of chaotic attractor\(^5\).

The effect of this occurrence on the investors’ behavior and therefore on issuer’s solvency are difficult to assess with any accuracy. To fix ideas, let us suppose that, perhaps as a result of an \textit{una tantum} intervention of the

\(^5\)We still have a stable 2-cycle around \( E_1 \) but we don’t show it.
Central Bank, the initial value of $p$ is located near the “good” equilibrium price, $\bar{p}_2$. In the case under consideration, $p$ will oscillate in a complex way, taking values above as well as below $\bar{p}_2$, without ever leaving a more or less large interval $I$ around it. Two elements are important here: the size of $I$, (a measure of the amplitude of oscillations) and the probability with which the price takes “good” or “bad” values (above or below $\bar{p}_2$). More generally, we should expect a negative effect due to investors’ greater uncertainty and consequent greater perception of risk. An even worse case is depicted by Figure 6, showing a trajectory of $p$ that oscillates irregularly around the unstable equilibrium $E_2$ before converging to $E_1$.

[Figure 6 approximately here]

6.3 Only one equilibrium exists

The existence of multiple (in our case three) equilibria does not necessarily follow from the assumption of the existence of a upward–sloping tract of the demand curve. Figure 7 illustrate the case for which only the “bad” equilibrium $E_1$ exists and it is stable.

[Figure 7 approximately here]

In this case, even if the initial value of the bond price (yield) is sufficiently high(low) to guarantee solvency, a perverse dynamics will follow with price converging to $E_1$, and presumably leading to financial collapse.

Next, consider the situation when only the “good” equilibrium exists. If it is globally stable, the the bond price converges to $E_2$ and its dynamics have a happy end. But this need not be the case. Figure 8 illustrates the case for which the unique equilibrium $E_2$ is unstable and there are wide oscillations of the bond price (and its yield) around it.

[Figure 8 approximately here]

The negative effects of this situation on the issuer’s solvency are no better than those considered above, because now the bond price (and the associated yield) oscillates wildly and it often and repeatedly takes values incompatible with solvency.
6.4 Intermittency and chaos

We conclude the (non-exhaustive) list of our examples with a curious but by no means implausible case, known in the theory of dynamical systems as “intermittency”. Strictly speaking, the demand and supply curves intersect only once at $E_2$, but are very close for a lower value of $p$, call it $p_k$. $E_2$ is unstable and the dynamics around it is characterized by wide, irregular oscillations. However, from time to time, $p$ moves close to $p_k$, in correspondence to which $ED(p_k)$ is very small and consequently $p$ changes very slowly, all the more so the closer to each other are $D(p_k)$ and $S(p_k)$. We could say that $p_k$ is “almost an equilibrium”. The resulting dynamics, showed by Figure 9, is puzzling: it is an alternation of seemingly chaotic oscillations and more or less long period of calm, around $p_k$. Prima facie, one would say that the dynamics is subject to external, random shocks but of course there are none in our model. This is indeed a very bad situation for the bond issuer’s solvency, all the more so the lower the level of $p_k$.

[Figure 9 approximately here]

7 Conclusions

In this paper we have shown that, even for an heroically simplified representation of a sovereign bond market, things may be be significantly different from and greatly more complicated than we are often led to expect.

First of all, even in the most favorable circumstances – a single equilibrium and an everywhere downward-sloping excess-demand function – the equilibrium may not be stable and, transients apart, the state variables may converge to a cycle situated more or less far from equilibrium. Thus, even if this unique equilibrium is “good” in the sense that the corresponding price(yield) is compatible with a country’s solvency, this is not necessarily true for the average price(yield).

Second, if the single equilibrium is unstable and, due to the presence of upward-sloping tracts of the demand curve, the controlling map is non-monotonic, the asymptotic dynamics of the bond’s price (and the corresponding yield) may be very complicated (cyclical with different periods or even chaotic).

Third, the “good” equilibrium may be unstable or nonexistent.

Fourth, in the multiple equilibria case, many different, more or less complicated dynamics may occur. The existence of three equilibria does not necessarily imply the commonly adopted scenario with two stable, “good”
and “bad”, equilibria and an intermediate one, always unstable dividing the price space into two zones characterized, respectively by a road to solvency and a road to financial collapse. We have shown that this need not be the case. Either the “good”, or the “bad” equilibrium, or both, may be unstable and complicated dynamics are possible.

Fifth, except in the simplest more favorable cases, an external push, moving the bond’s price up (the bond’s yield down), close as we wish to its “good” equilibrium value may not be sufficient to stabilize the market, and sustainability may require a continuous and much more articulate support.

Of course, the classes of models discussed in this paper, standard or otherwise, do not tell all the story. They are far too simple and leave out many important factors contributing to the dynamics of bond market and sovereign countries’ solvency. We mention only two of them. First, in real markets there are external shocks, random or not, affecting both fundamentals and, most importantly at least in the short run, investors’ expectations and decisions. Secondly, the macroeconomic quantities defining the solvency condition (1) are neither constant nor independent of one another. Specifically, there exist complex and crucial interdependencies among budget primary deficit $s$, the rate of growth $g$, and bond yield $r$. The recent, hot debates among macroeconomists and policy makers on the effect of austerity on growth (and, via the “fiscal multiplier”, on the debt/GDP ratio) are relevant here. Moreover, a country’s capability of sustained growth affects investors’ assessment of the risk of its debt and thereby the bonds’ price(yield). The latter affects the budget deficit and, other things being equal, the country’s ability to realize a sufficiently large primary surplus.

All this, however, does not imply that the argument developed so far is valueless. Unless we are able to derive precise, cogent conclusions from a simple but rigorously defined and not unreasonable model, we can hardly hope to effectively tackle a more realistic, but infinitely more complex one.

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Figure 1: A graphical representation of the standard model
(a) The demand and supply curves

(b) Convergence to equilibrium $E_2$

Figure 2: The pure standard case
(a) The demand and supply curves

(b) Oscillating Convergence to equilibrium $E_2$

Figure 3: The modified standard case
(a) The demand and supply curves

(b) A stable cycle–2 around equilibrium $E_2$

Figure 4: All equilibria unstable: periodic dynamics
(a) The demand and supply curves

(b) Chaotic dynamics around $E_2$

Figure 5: All equilibria unstable: complex dynamics
(a) The demand and supply curves

(b) Chaos around $E_2m$ convergence to $E_1$, “bad” equilibrium

Figure 6: Stable “bad”, unstable “good” equilibrium
(a) The demand and supply curves

(b) Convergence to the unique, stable equilibrium

Figure 7: Only a “bad” equilibrium exists

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Figure 8: Only the “good” equilibrium exists but it is unstable
(a) The demand and supply curves

(b) Coexistence of chaotic and quiet dynamics

Figure 9: Only one, “good” but unstable, equilibrium. Intermittency and chaos